Characterizing the Universe with N^{th}-Order Correlation Functions

Ashley J Ross, Robert J Brunner, Adam D Myers

What do we want?

What do we want?

- Quantitative measure of structure
- Comparable to models based on fundamental physics

What do we want?

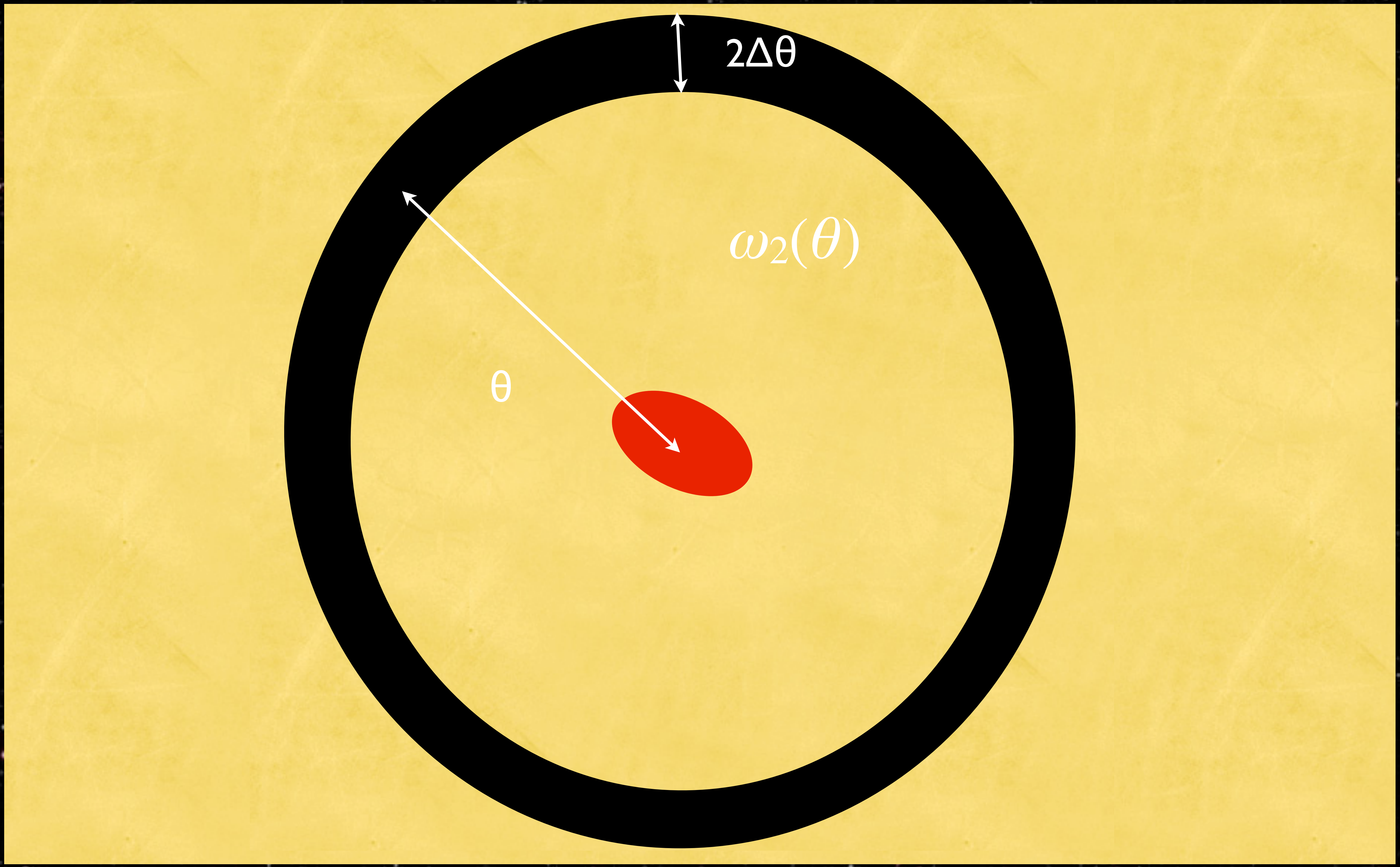
- Quantitative measure of structure
- Comparable to models based on fundamental physics
- →Correlation Functions
- Key component of concordance cosmology
- In future — nature of dark energy, primordial non-Gaussianity, galaxy formation and evolution

Outline

- N^{th}-order correlation functions
  - Definition
  - Methods of measurement
  - Methods of interpretation
- Selected results
  - Measuring σ_8 with LRGs
  - N^{th}-order clustering by color & redshift
- Future prospects

N^{th}-Order Correlation Functions

- *N*-point angular correlation function, ω_N:
  - Given a random object in a location, $\omega_2(\theta)$ likelihood that another object will be found at distance $\theta \pm \Delta\theta$
  - $\omega_3(\Phi, \theta_1, \theta_2)$ likelihood of finding object at distance $\theta_1 \pm \Delta\theta$ and another object at $\theta_2 \pm \Delta\theta$ with vertexes making angle $\Phi \pm \Delta\Phi$


2Δθ
$\omega_2(\theta)$
θ

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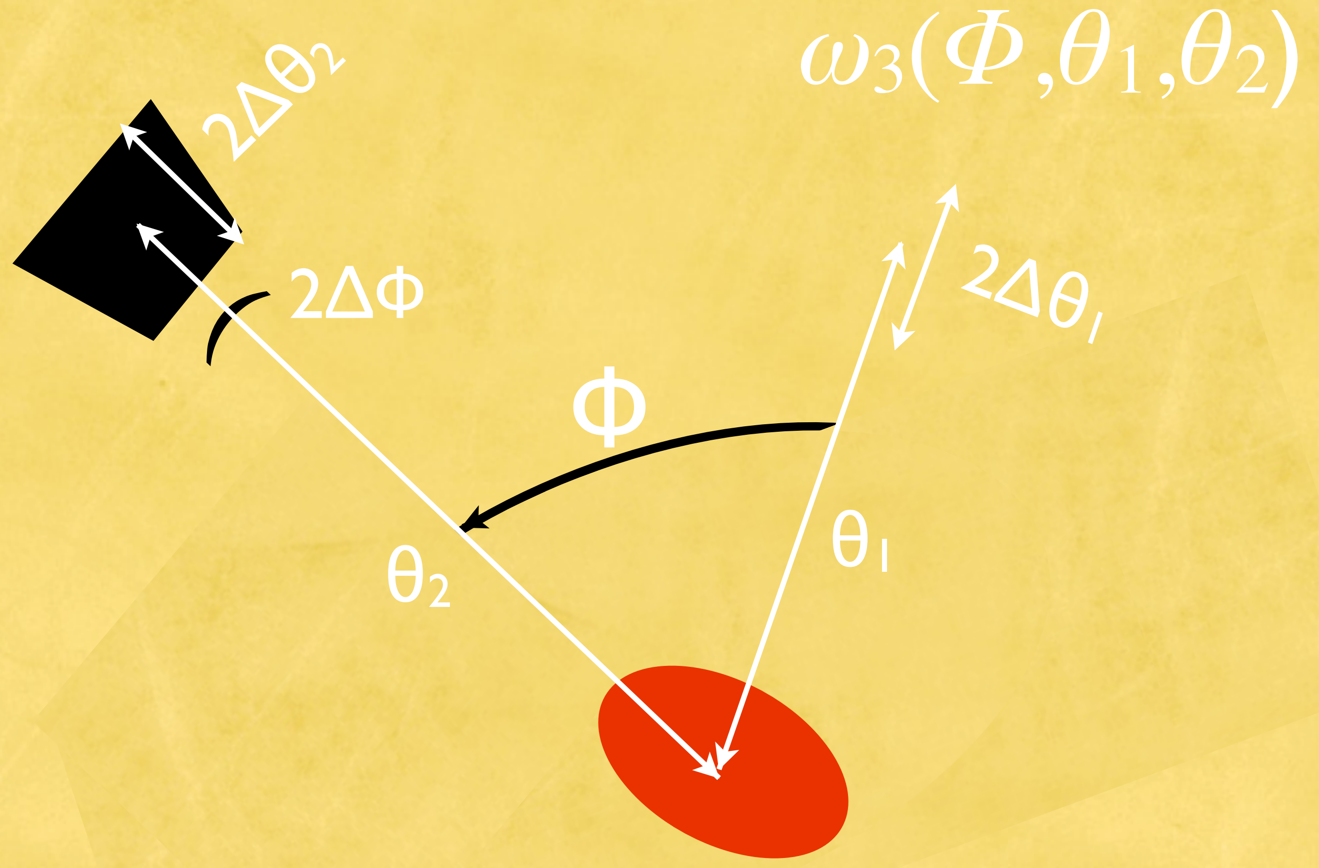
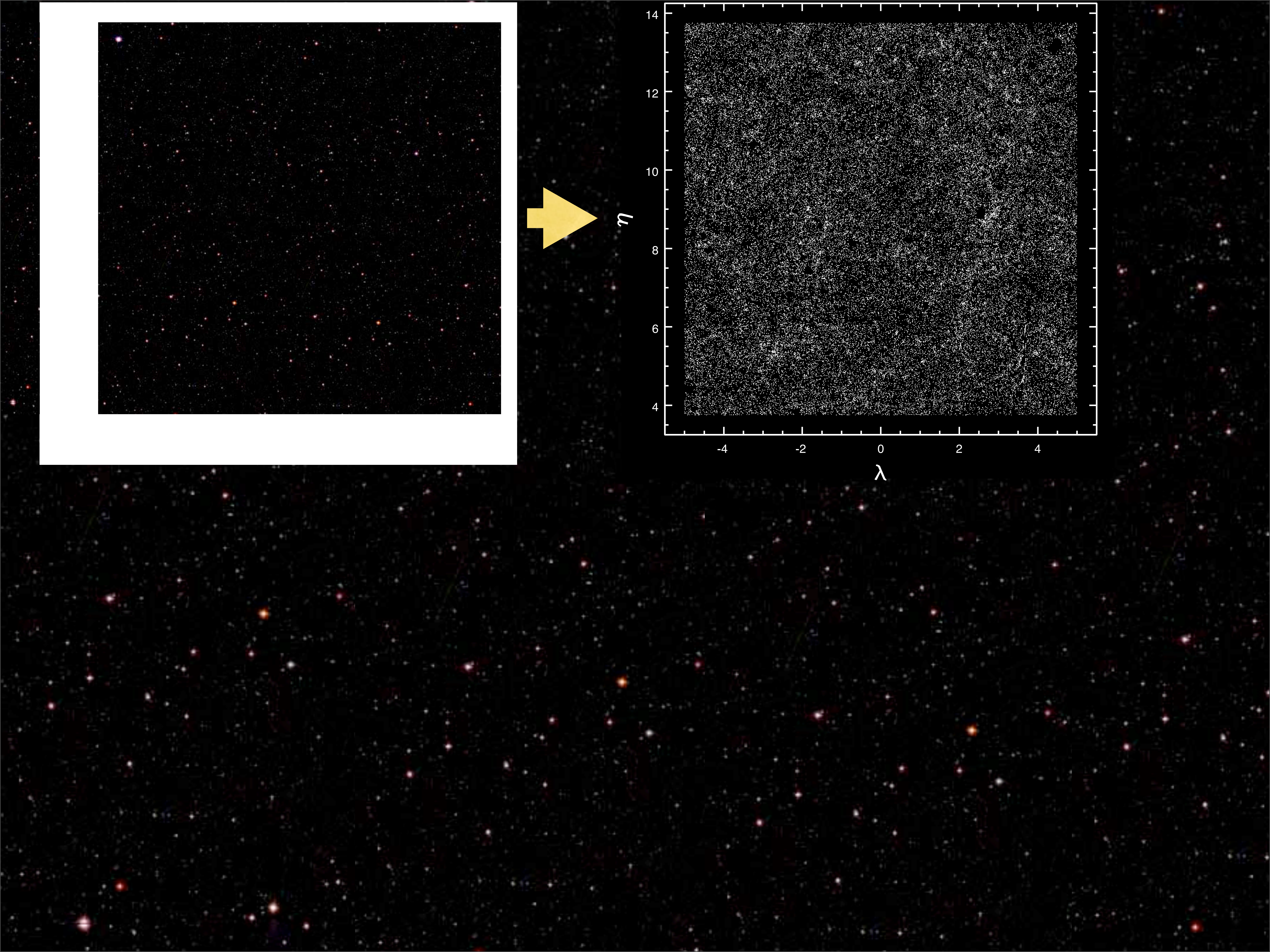
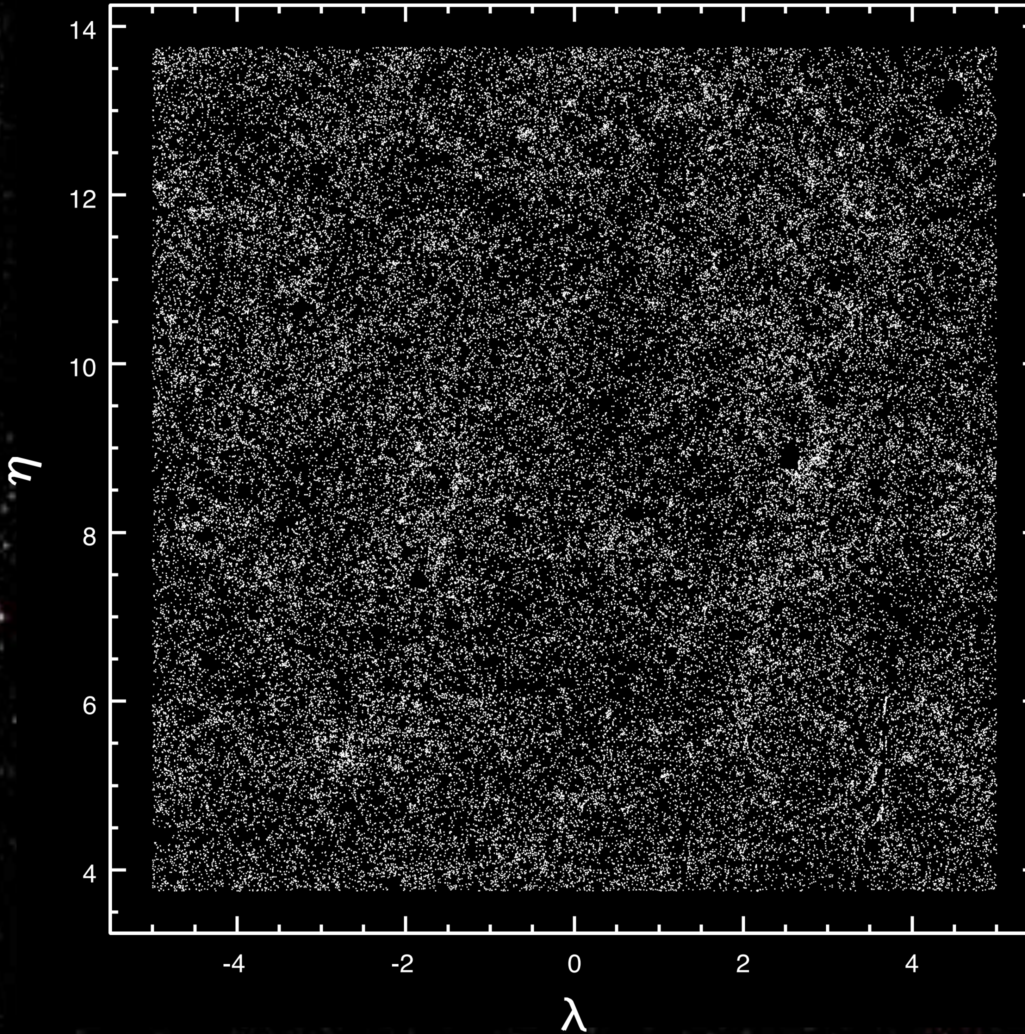
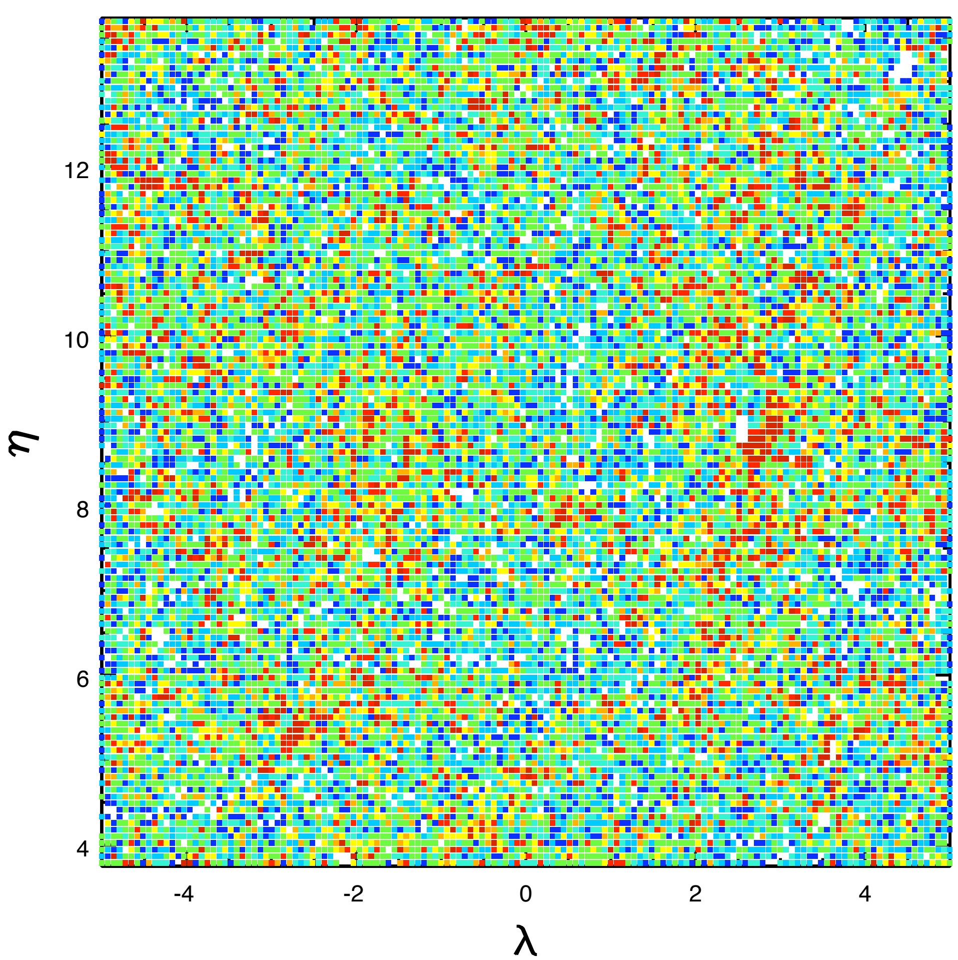

$\omega_3(\Phi, \theta_1, \theta_2)$
$2\Delta\theta_2$
$2\Delta\Phi$
$2\Delta\theta_1$
Φ
θ_1
θ_2

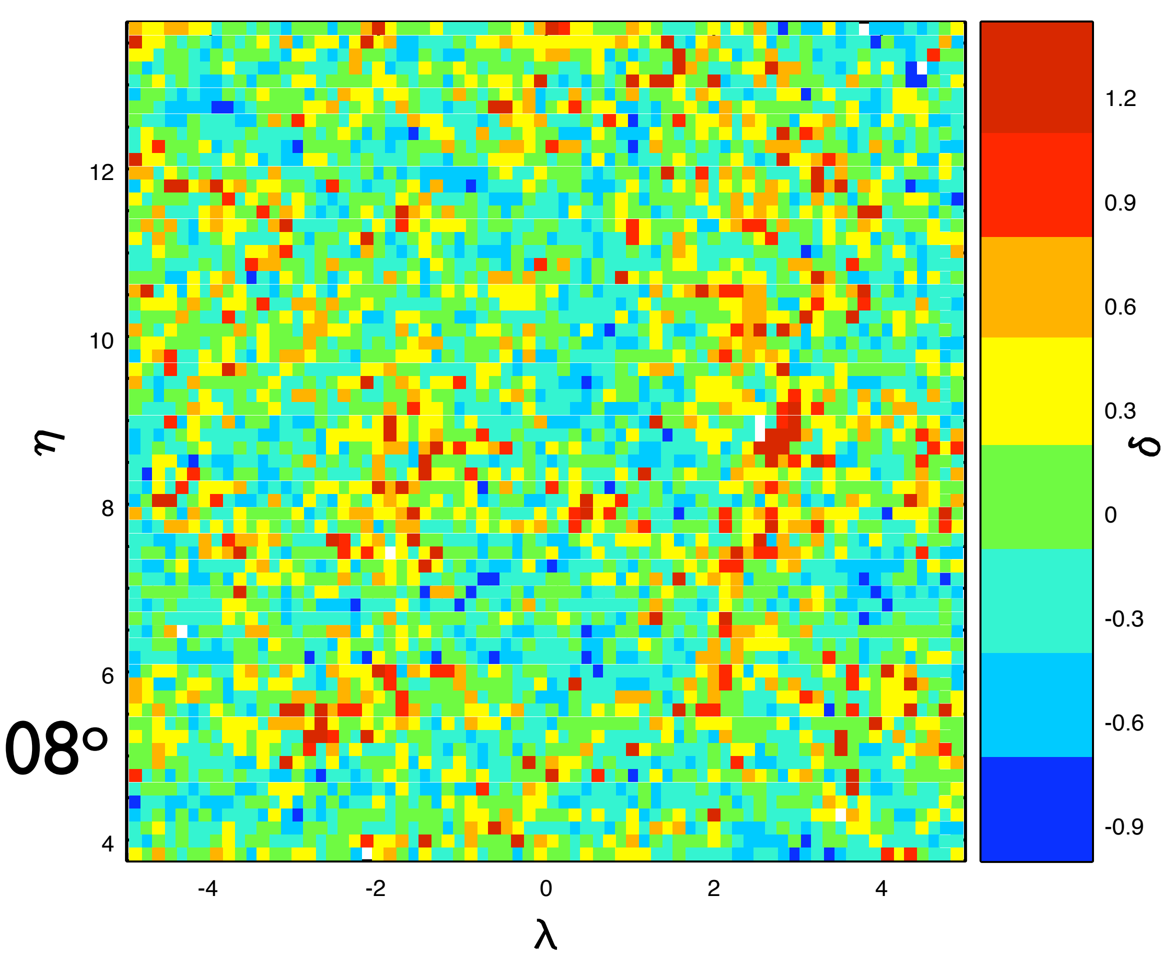

14
12
10
8
6
4
η
-4
-2
0
2
4
λ





~0.04°

~0.08°

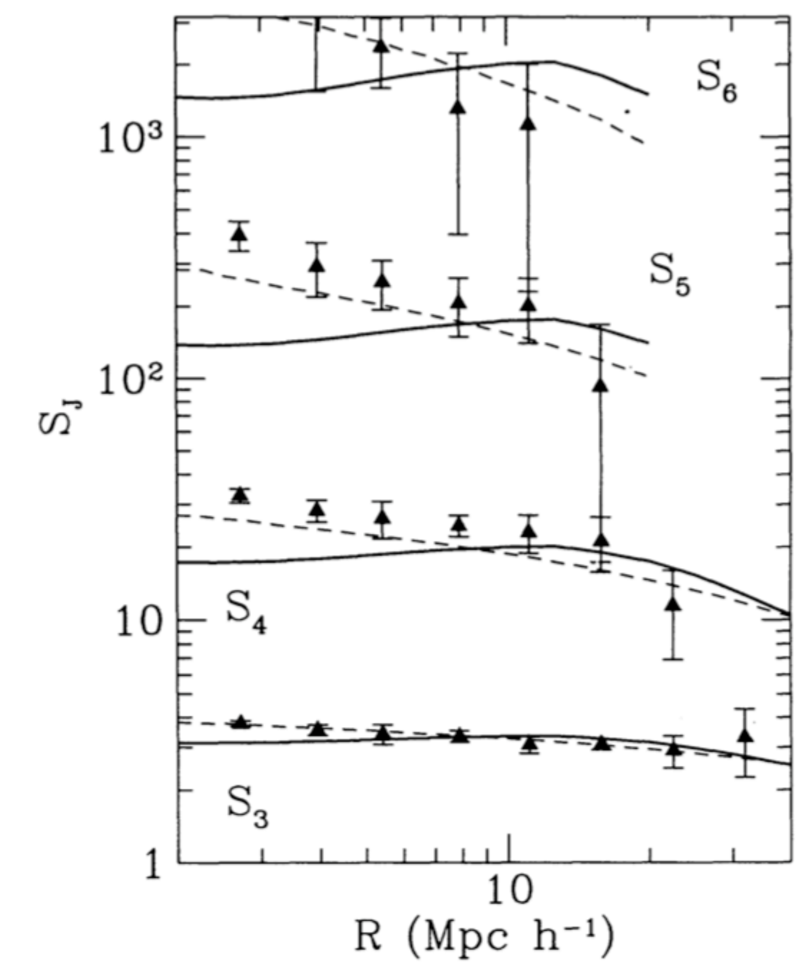


N^{th}-Order Correlation Functions

- Overdensity, $\delta = \frac{N}{\langle N \rangle} - 1$
- *N*-point angular correlation function, ω_N:
  - $\omega_2(\theta) = \langle \delta_i \delta_j \rangle$, $\omega_3(\phi, \theta_1, \theta_2) = \langle \delta_i \delta_j \delta_k \rangle$, ...
- *N*-point *area averaged* angular galaxy correlation function, $\bar{\omega}_N$:
  - $\bar{\omega}_N(\theta) = \langle \delta^N \rangle_c$, $s_N(\theta) = \frac{\bar{\omega}_N}{\bar{\omega}_2^{N-1}}$
- For gaussian density field, higher-order terms vanish

Higher-Order Correlations

- Gaussian or nearly Gaussian density field evolves under gravitational collapse
- non-linear PT with CDM model predicts hierarchy in S_N
- Measurements generally agree with theory



Gaztañaga & Frieman (1994)

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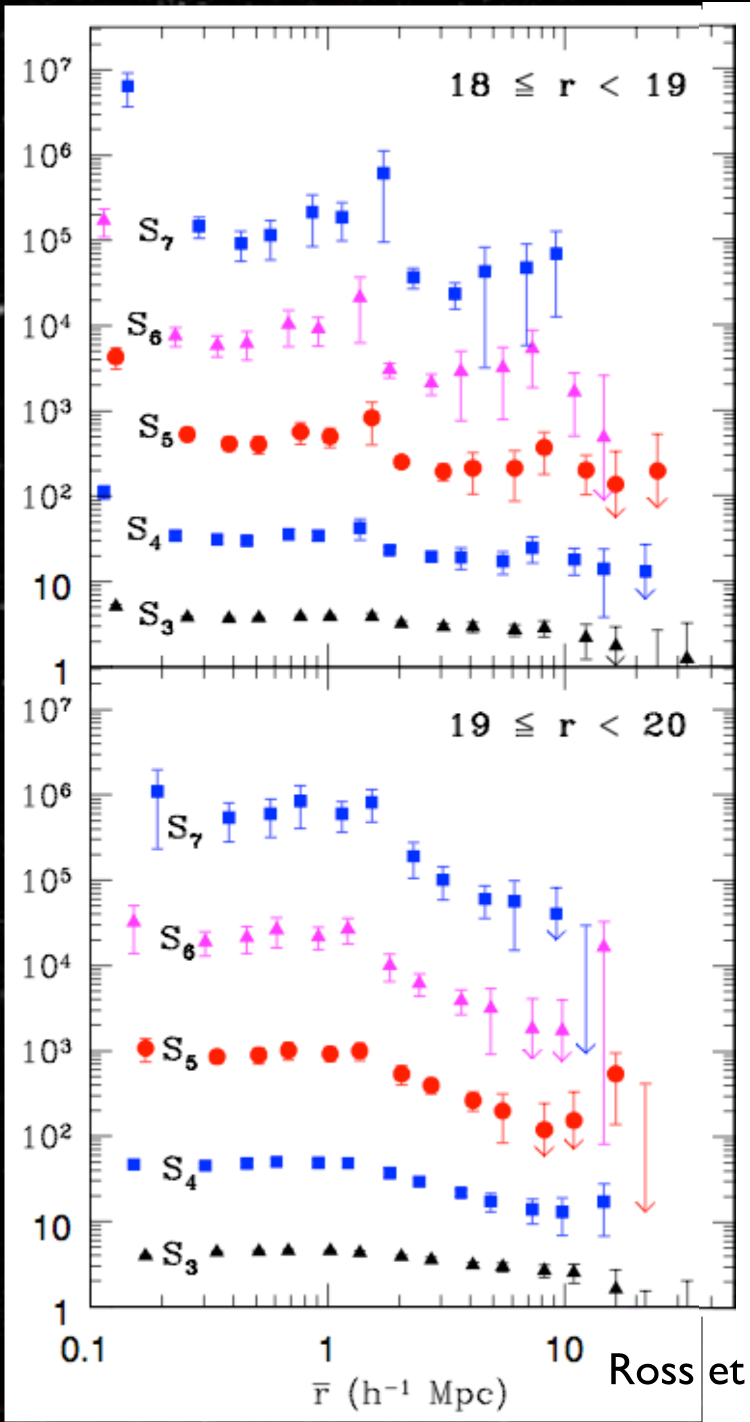


Ross et al. (2006)

Modeling ω_N

- Given Smith et al. (2003) $P(k)$:

$$\omega_{2,DM}(\theta) = \frac{\pi}{c}\int_0^\infty \left(\frac{dN}{dz}\right)^2 H(z)dz \int_0^\infty P(k,z)J_0[k\theta\chi(z)]kdk$$

$$\bar{\omega}_{2,DM}(\theta) = \frac{\pi}{c}\int_0^\infty \left(\frac{dN}{dz}\right)^2 H(z)dz \int_0^\infty P(k,z)W_{2\mathrm{D}}^2[k\theta\chi(z)]kdk$$

$$\bar{\omega}_{3,DM}(\theta) = 6\left(\frac{\pi}{c}\right)^2 \int_0^\infty \left(\frac{dN}{dz}\right)^3 H^2(z)dz \times \left[\frac{6}{7}\left(\int_0^\infty kdkP(k,z)W_{2\mathrm{D}}^2[k\theta\chi(z)]\right)^2 \right.$$
$$\left. +\frac{1}{2}\int_0^\infty kdkP(k,z)W_{2\mathrm{D}}^2[k\theta\chi(z)]\int_0^\infty k^2dkP(k,z)W_{2\mathrm{D}}[k\theta\chi(z)]W_{2\mathrm{D}}'[k\theta\chi(z)]\right]$$

$$W_{2\mathrm{D}}(x) = 2\frac{J_1(x)}{x}$$

- Can also use HOD to find $P_{gal}(k,z)$

Bias

- Bias relates galaxy clustering to dark matter clustering

- Local bias model:

$$\delta_g = F(\delta_{\rm DM}) \Rightarrow \delta_g = b_1\delta_{\rm DM} + 0.5 b_2 \delta_{\rm DM}^2 + O(\delta_{\rm DM}^3)$$

$$\delta_{\rm DM} = b_1^{-1}\delta_g - 2b_1^{-3} b_2 \delta_g^2 + O(\delta_g^3)$$

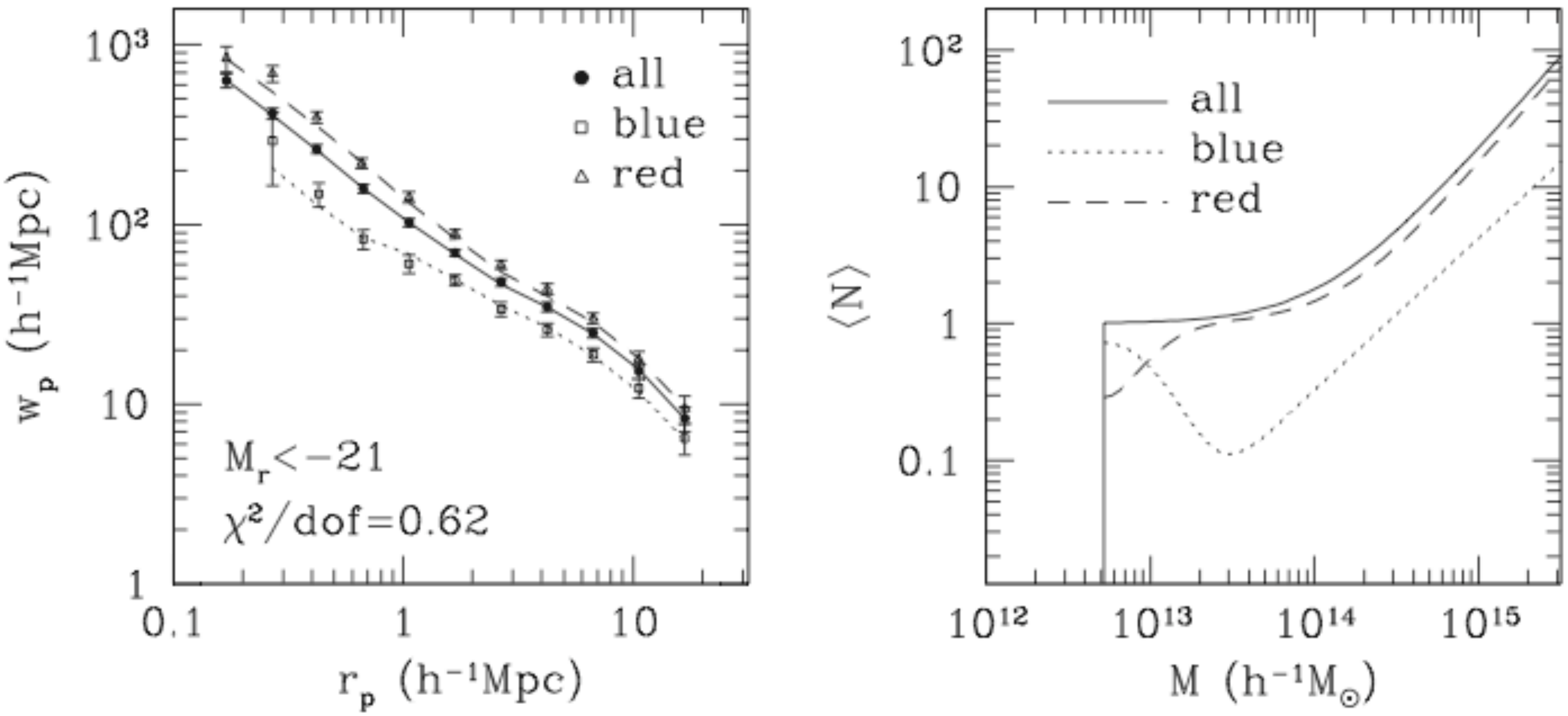
- For $r_{\rm eq} \gtrsim 10h^{-1}{\rm Mpc}$ and $c_2 = b_2/b_1$:

$$\omega_2 \cong b_1^2 \omega_{2,{\rm DM}}$$

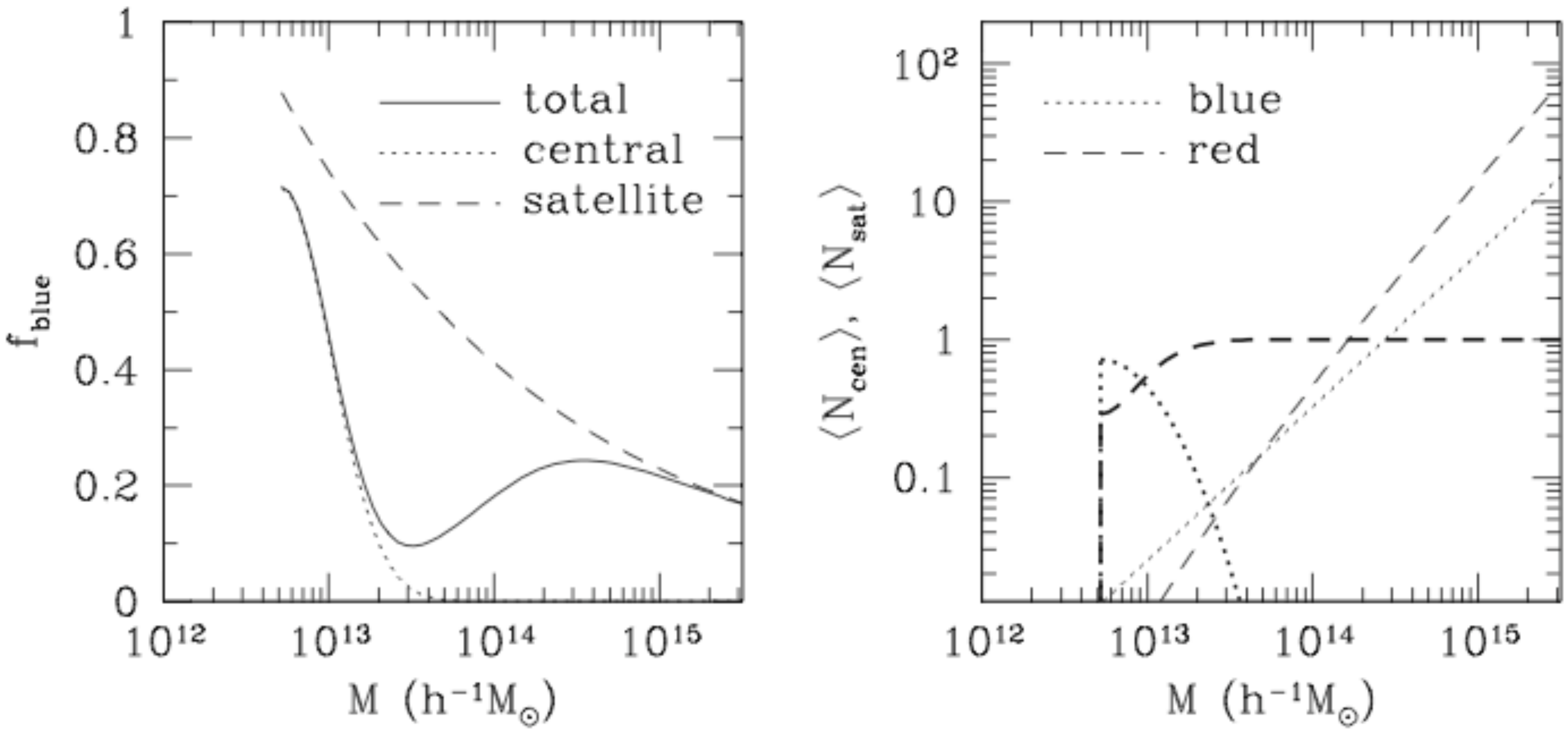
$$s_3 \cong b_1^{-1}\left(s_{3,{\rm DM}} + 3c_2\right)$$

Correlation Functions

- Two main pursuits:
  1) Study galaxies themselves
  2) Measure cosmological parameters



Zehavi et al. (2005)



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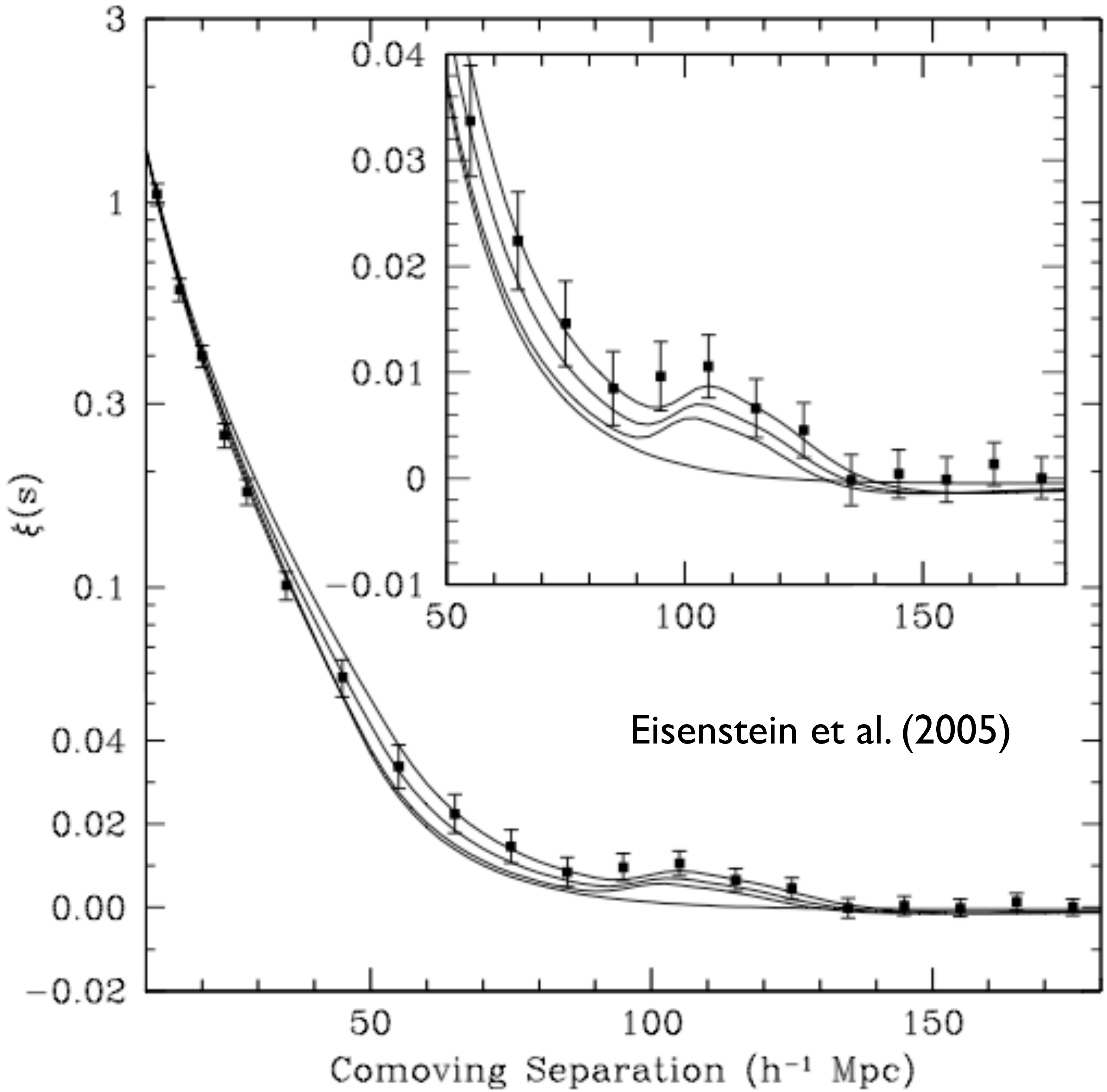

3
1
0.3
0.1
0.04
0.02
0.00
−0.02
ξ(s)
50
100
150
Comoving Separation (h⁻¹ Mpc)
0.04
0.03
0.02
0.01
0
−0.01
50
100
150
Eisenstein et al. (2005)

Correlation Functions

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Correlation Functions

- Two main pursuits:
  1) Study galaxies themselves
  2) Measure cosmological parameters
- Doing one usually requires assumptions about the other
  - Mass scale of HOD is dependent on cosmology
  - LRG analysis requires knowledge of large-scale clustering vs. matter
- Higher-order correlations provide extra constraints, allowing less assumptions, more self-consistent measurements

σ_8

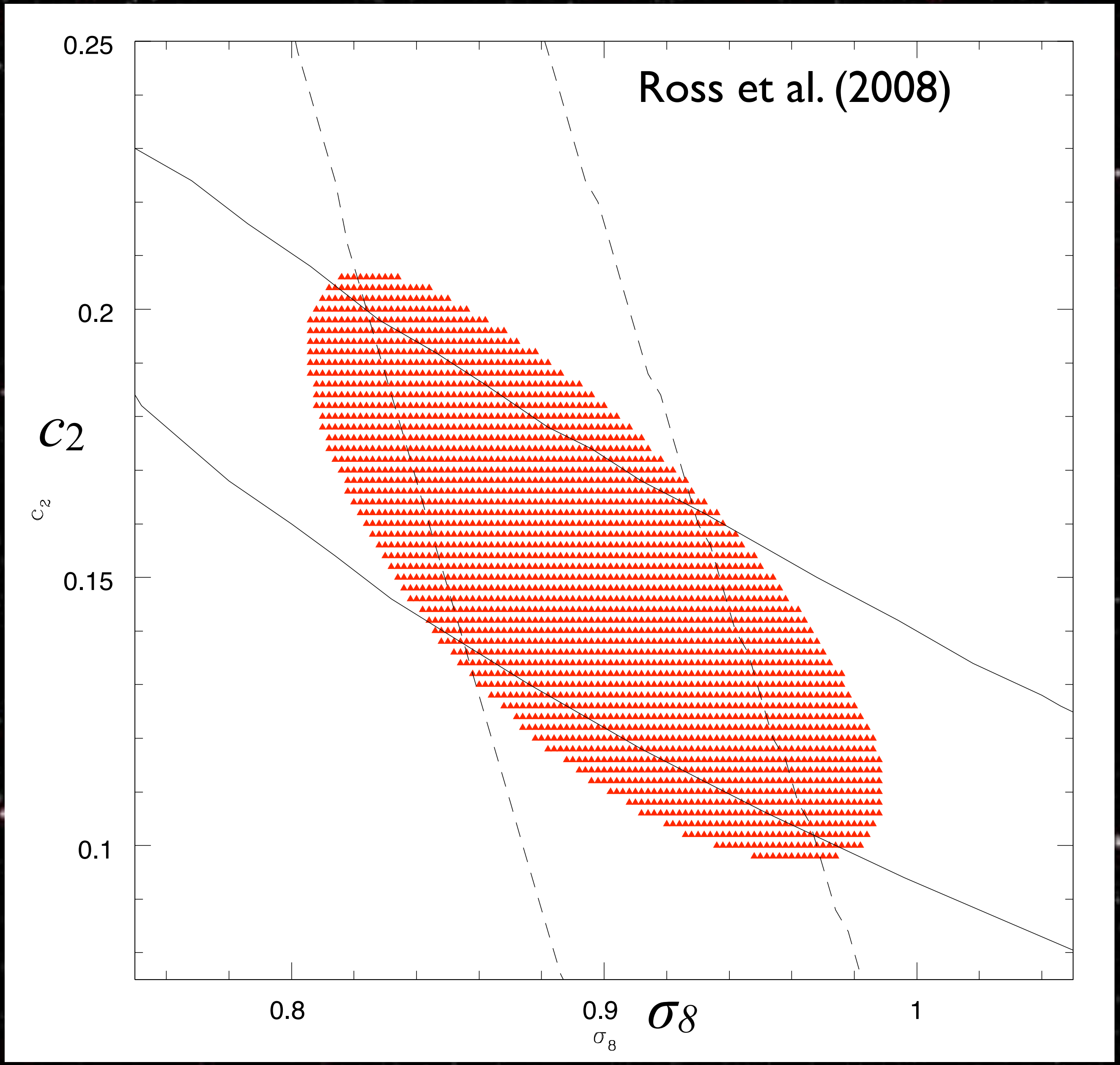
- σ_8 is the rms mass fluctuation at $8\ h^{-1}$ Mpc
- $\delta_{DM} \propto \sigma_8$ so $b_1 \propto 1/\sigma_8$
- This makes it nuisance parameter for 2-point measurements
- WMAP: $\sigma_8 = 0.92 \pm 0.10 \rightarrow 0.744^{+0.05}_{-0.06} \rightarrow 0.796 \pm 0.036$

Measuring σ_8

- Adding in 3-point measurements offers extra constraint and thus ability to calculate σ_8
- One method:
  - Measure s_3 for galaxies, determine $c_2(\sigma_8)$
  - Turn δ_g to δ_{DM} with assumed b_1 and b_2, measure *corrected* ϖ_2, match to model $\varpi_{2,DM}$, yields separate $c_2(\sigma_8)$

Testing on Millennium Simulation

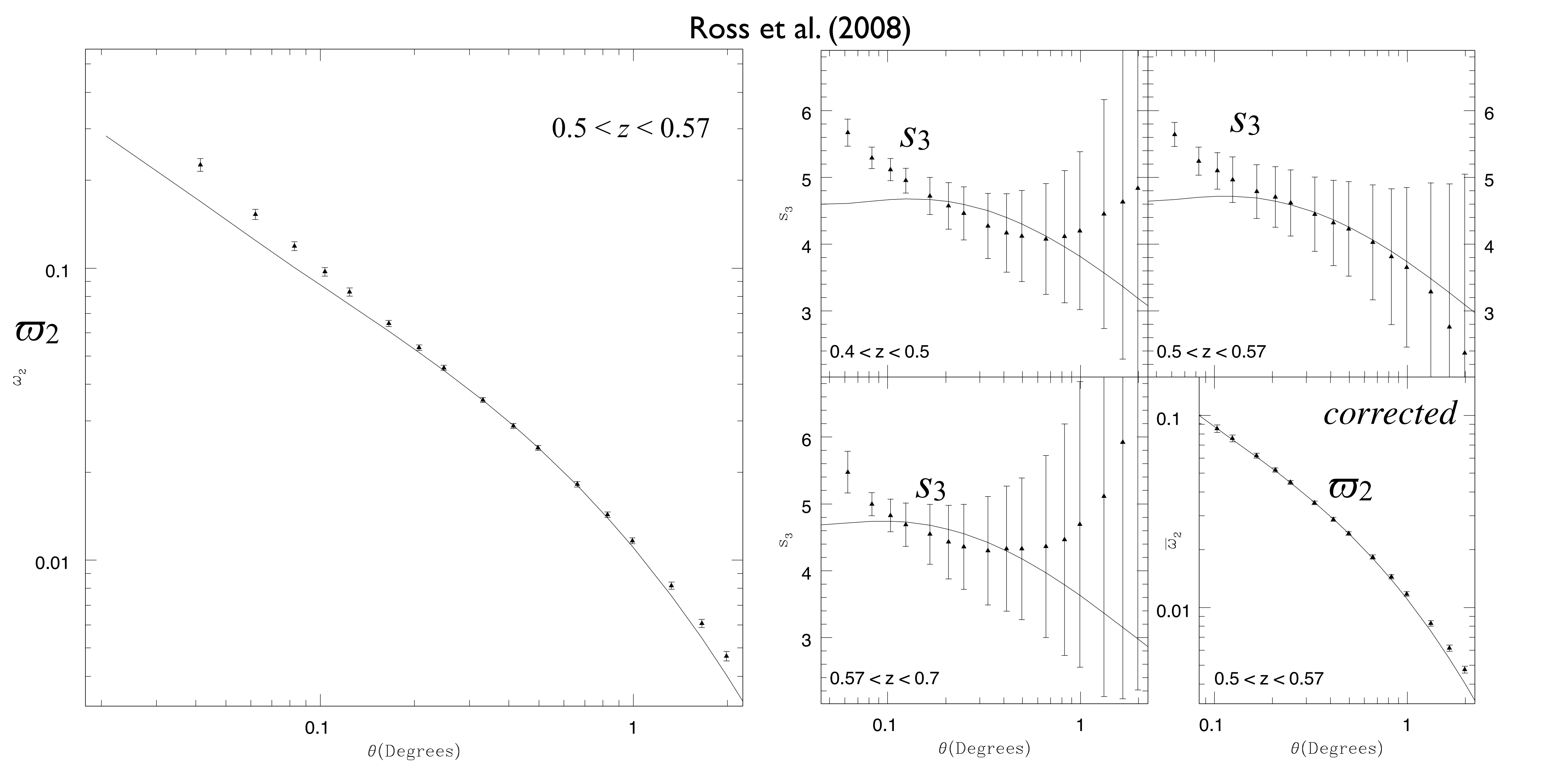
- $M_r < -23$ and B - $R > 1.4$ from Blaizot et al. (2005)
- Found $\sigma_8 = 0.898 \pm 0.062$
- (Input is $\sigma_8 = 0.9$)



SDSS LRG Catalog

- SDSS DR5 LRGs with MegaZ-LRG color cuts (Collister et al. 2007) and ANNz for photozs and star/galaxy separation
- Over 1.6 million LRGs with $0.4 < z < 0.7$ and median redshift of 0.52
- Split into three distinct redshift ranges with median redshifts of 0.47, 0.53, and 0.61

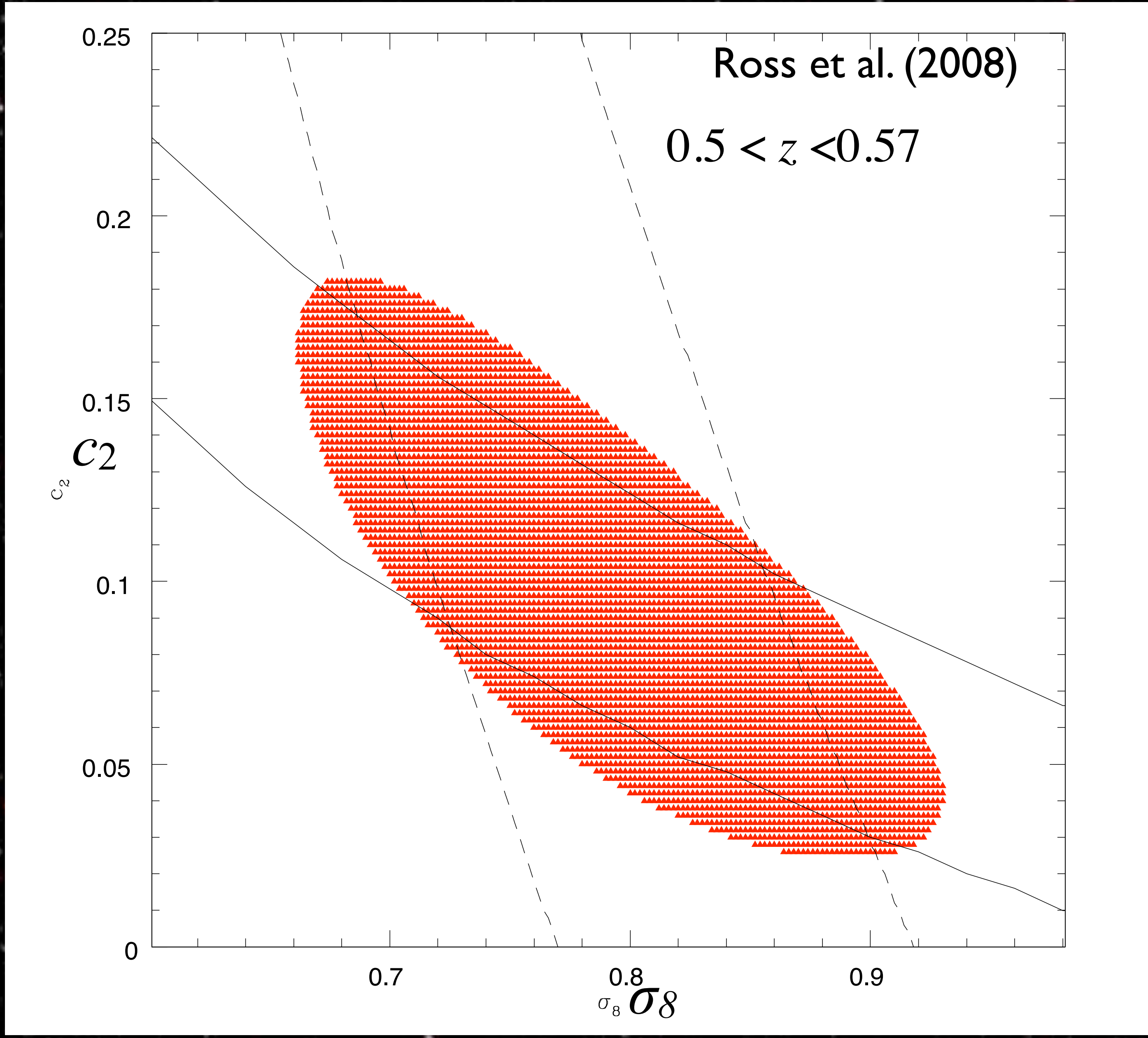
LRG Results



Ross et al. (2008)
0.5 < z < 0.57
ϖ2
ω2
0.1
0.01
0.1
1
θ(Degrees)
s3
6
5
4
3
0.4 < z < 0.5
s3
0.5 < z < 0.57
s3
0.57 < z < 0.7
corrected
ϖ2
0.5 < z < 0.57

LRG Results

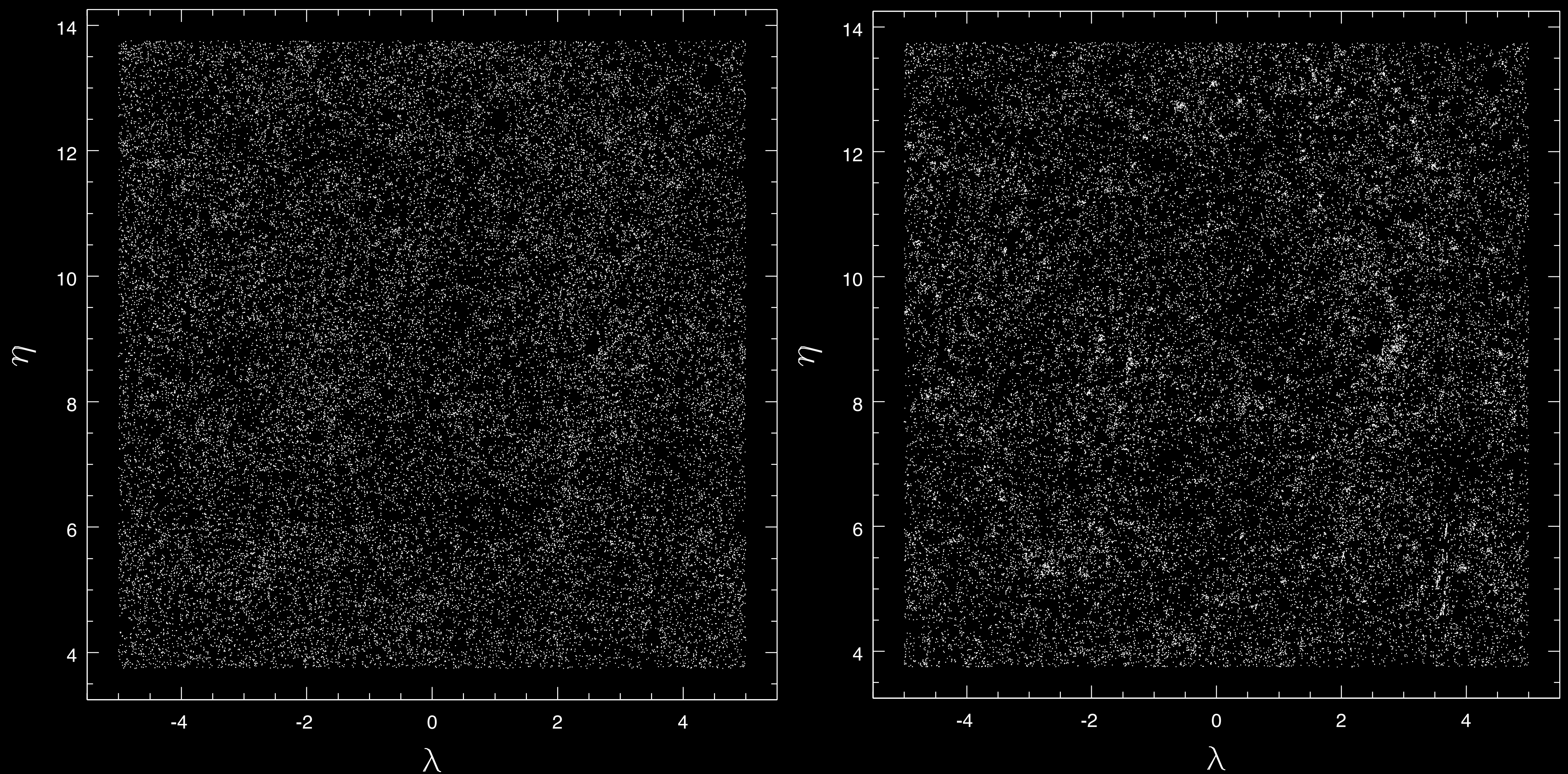
- Measured $\sigma_8 = 0.78 \pm 0.08$, 0.80 ± 0.09, and 0.80 ± 0.09
- Combine for $\sigma_8 = 0.79 \pm 0.05$
- Find $b_1 = 1.47 \pm 0.09$, 1.65 ± 0.09, 1.80 ± 0.10
- $c_2 = 0.09 \pm 0.04$, 0.09 ± 0.05, 0.09 ± 0.03



Clustering by Color

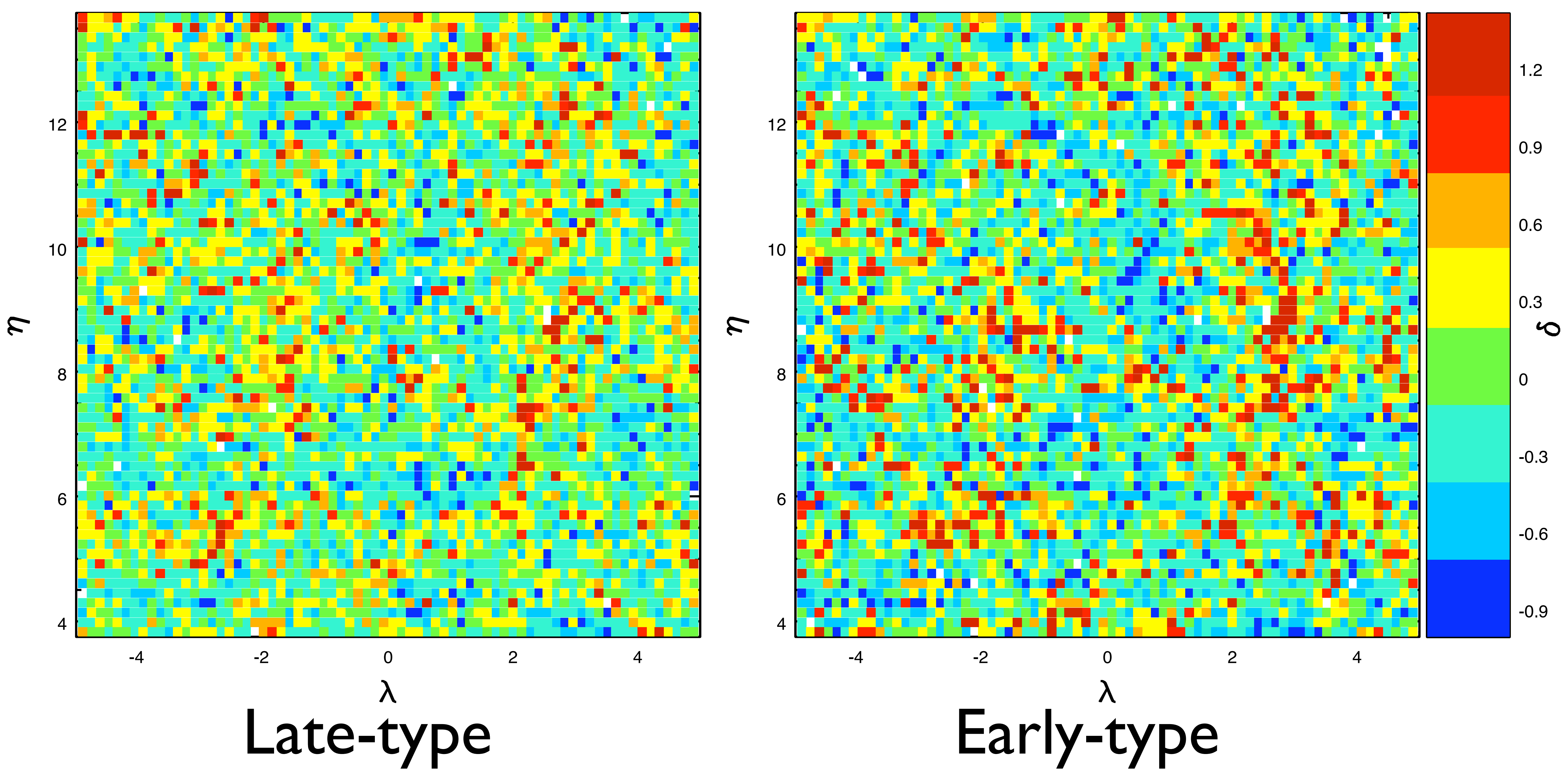
- Red (early-type) galaxies more clustered than blue (late-type) galaxies
    - Early-types found in clusters, more late-types in field
    - b_1, b_2 much larger for early-type
- Blue galaxies increasingly more clustered with redshift
    - Downsizing

Clustering by Color



Late-type Early-type

Clustering by Color



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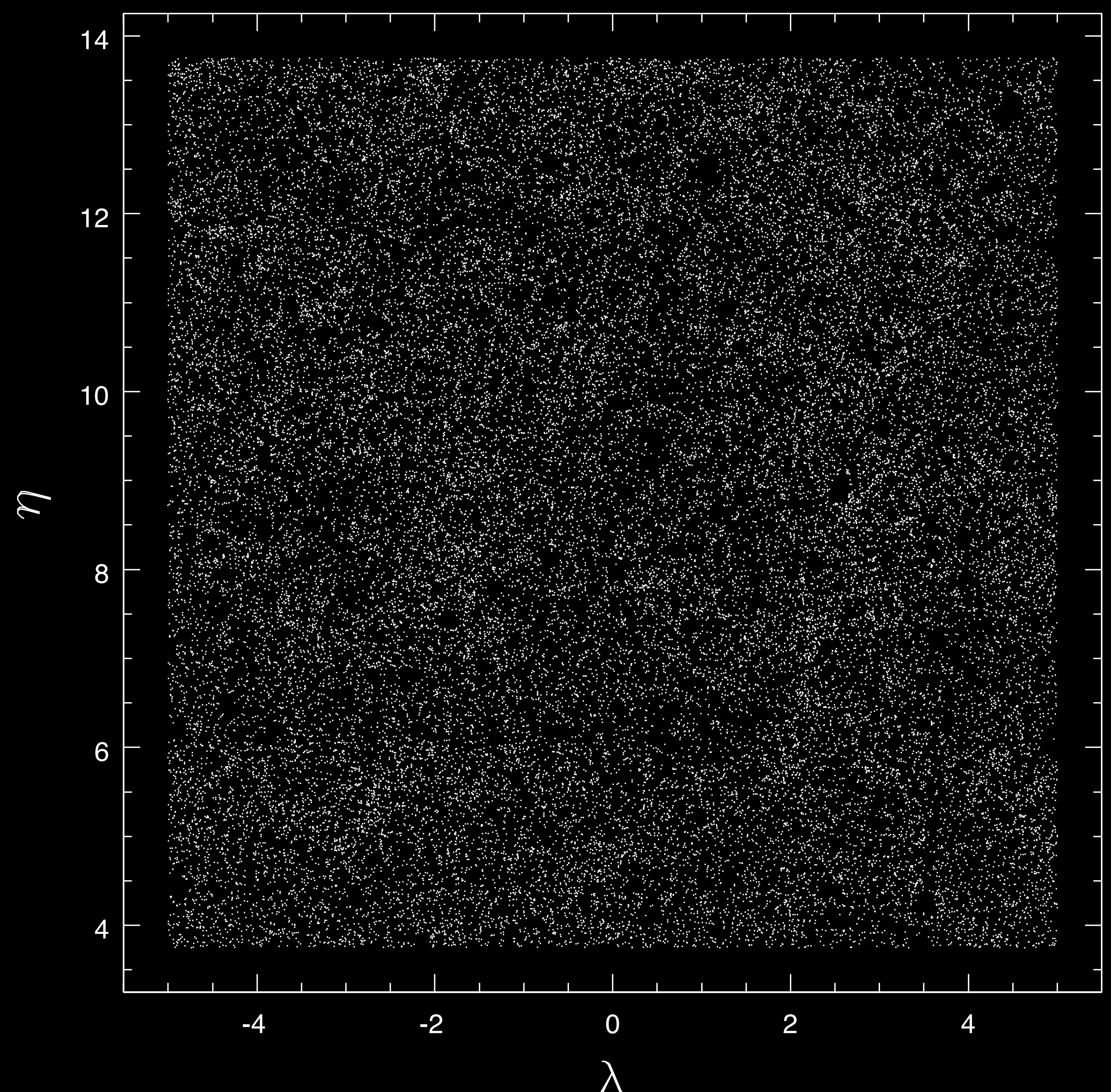
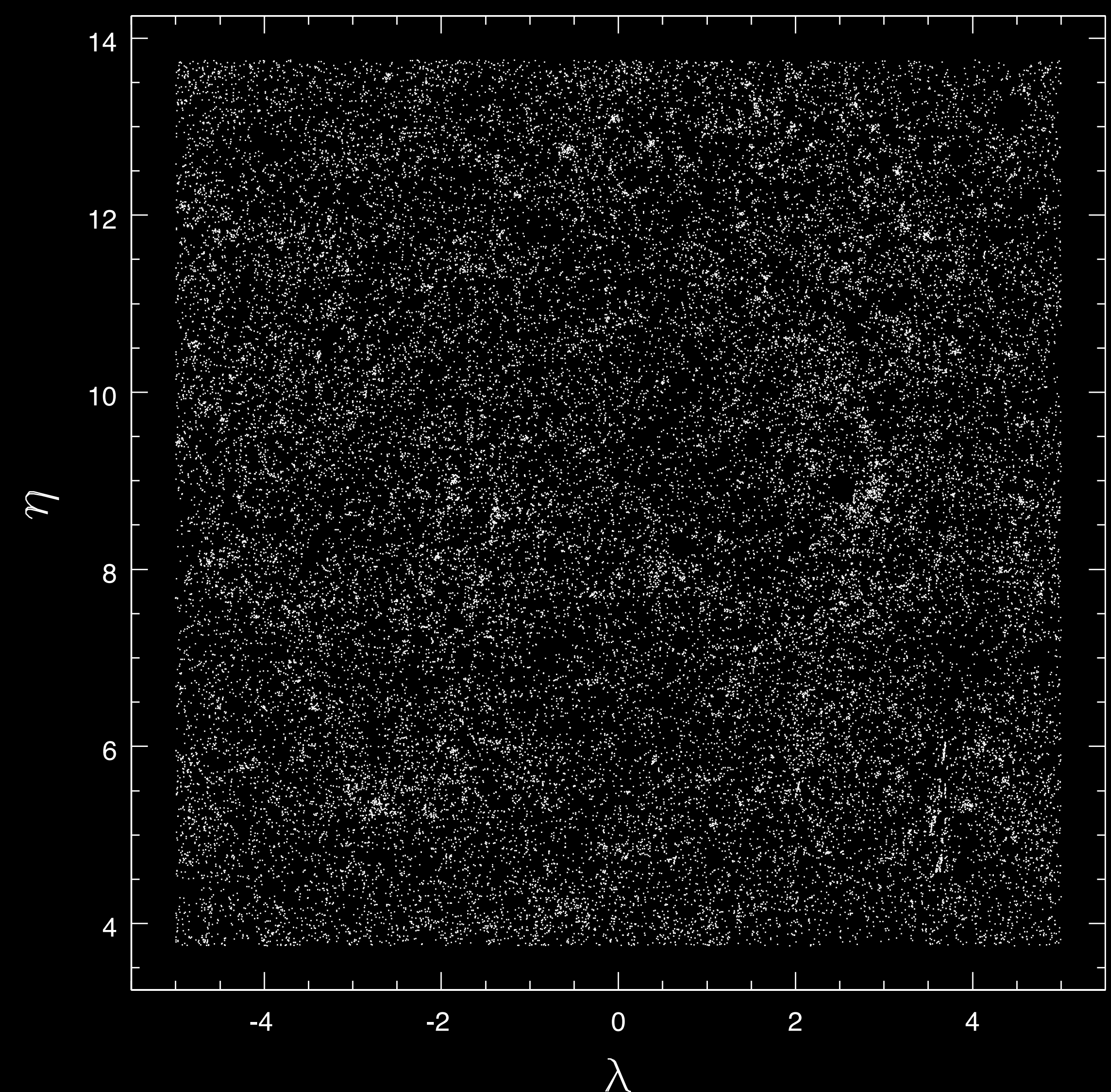


Late-type



Early-type

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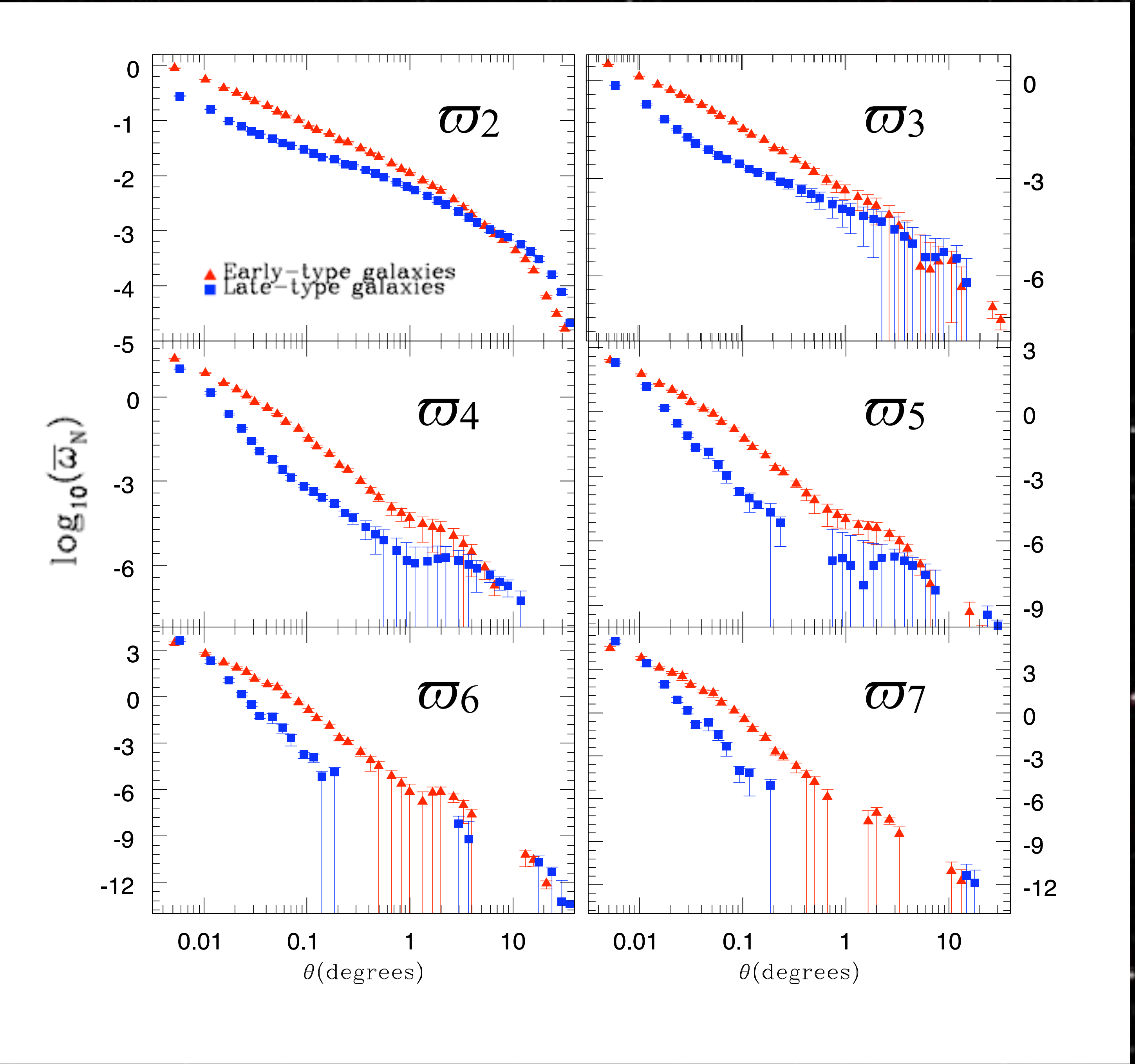
Higher-order Clustering by Color

- SDSS DR5 galaxies separated by u-r = 2.2
- Upturn in ϖ_N at small scales



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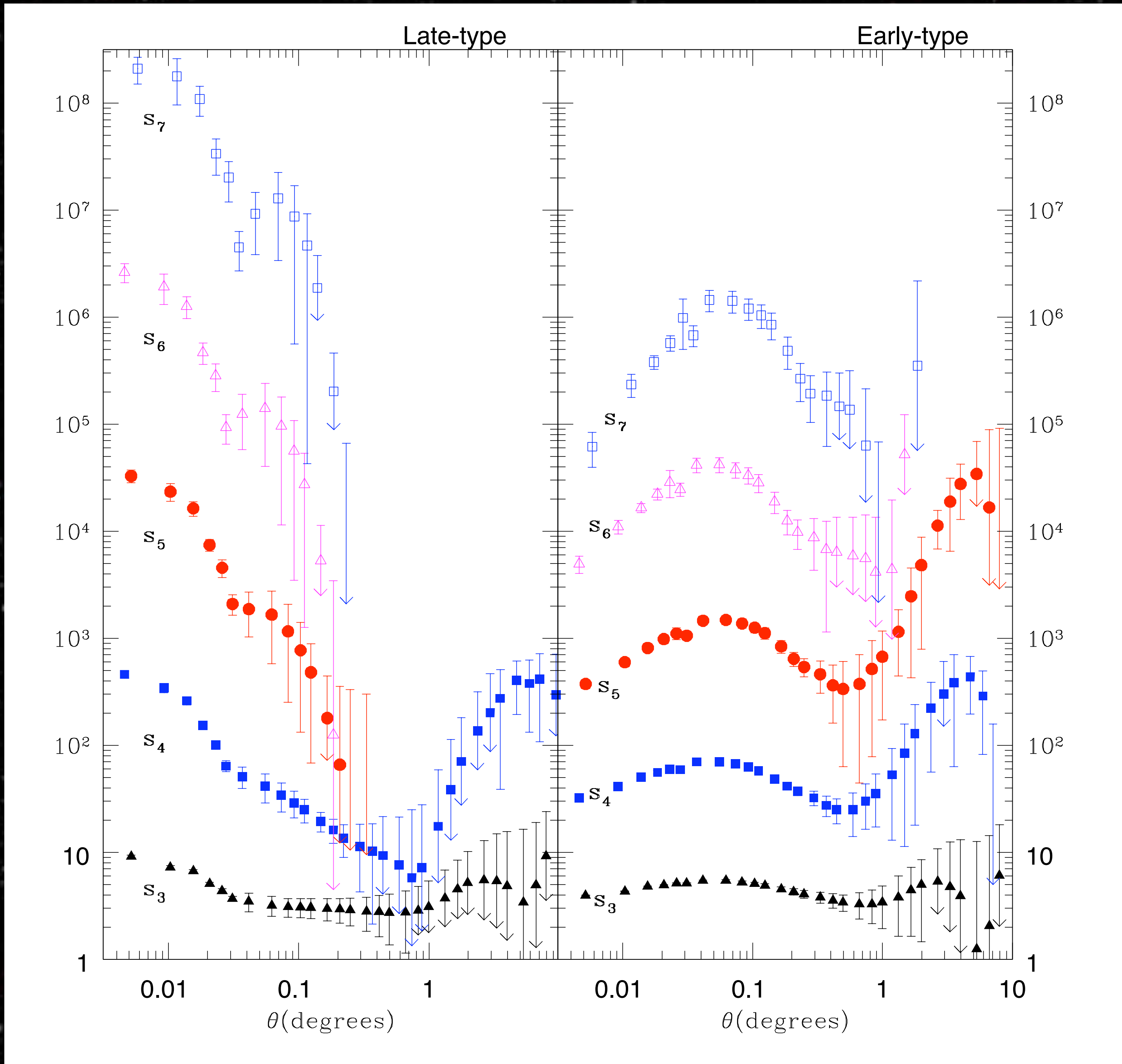
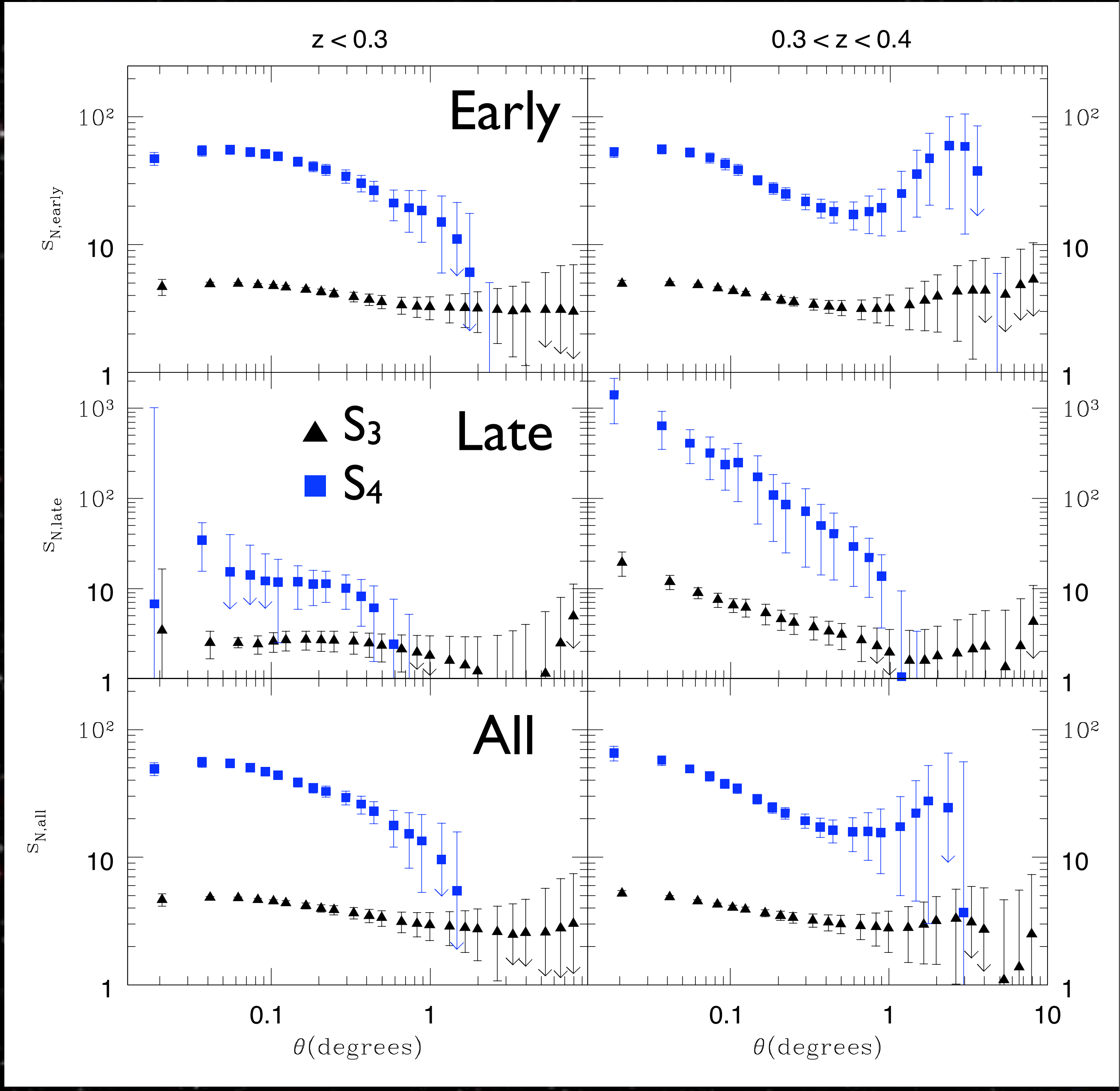


Redshift Evolution

- Galaxies taken from SDSS DR5 photoz $M_r < -20.5$
- Huge differences in small-scale clustering of late-types with z
- Bias changes from

$b_1, c_2 = 1.04 \pm 0.02, -0.83 \pm 0.21$

to $b_1, c_2 = 1.25 \pm 0.02, -0.38 \pm 0.30$



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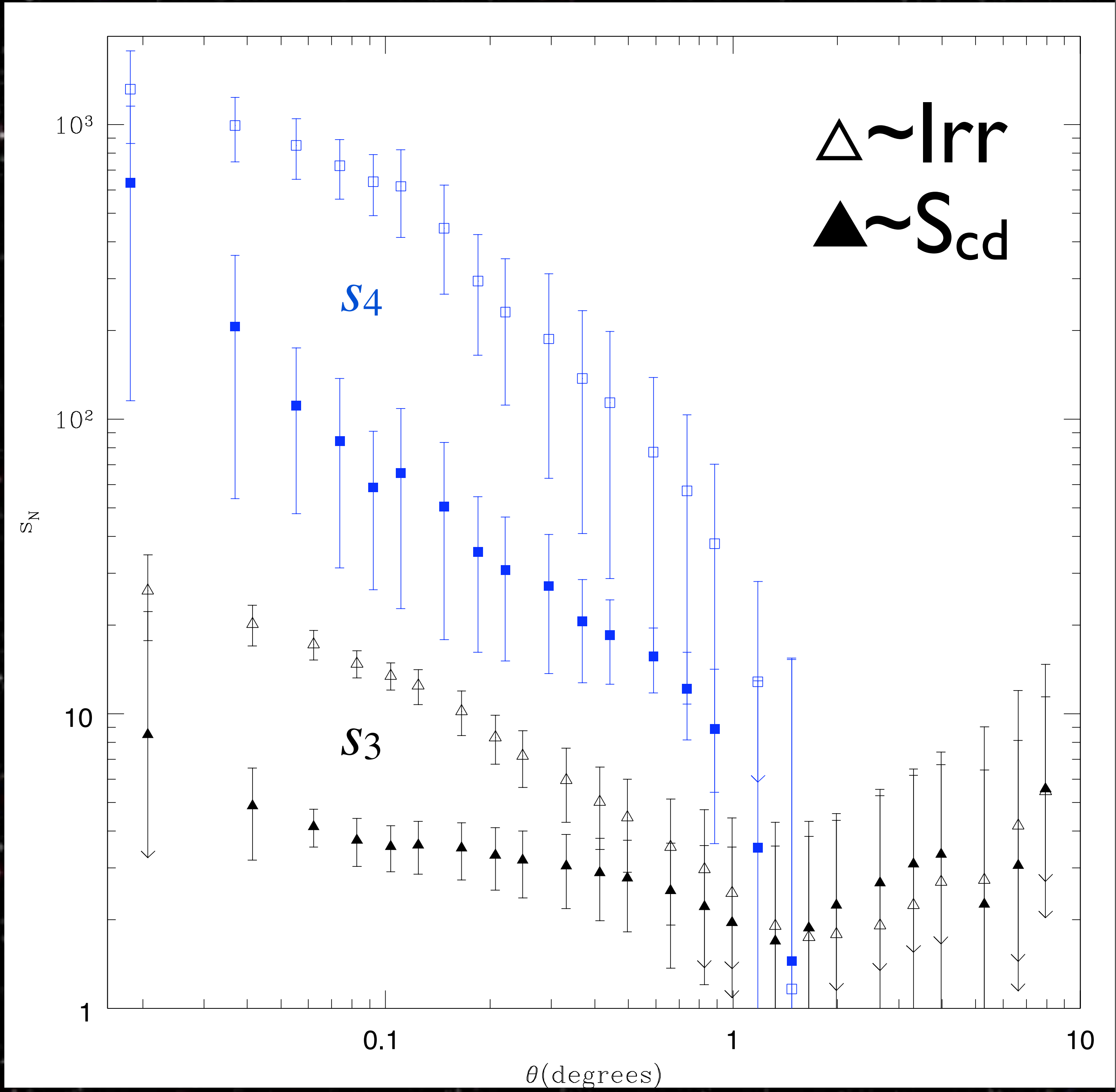


Late-type Galaxies

- HOD model:

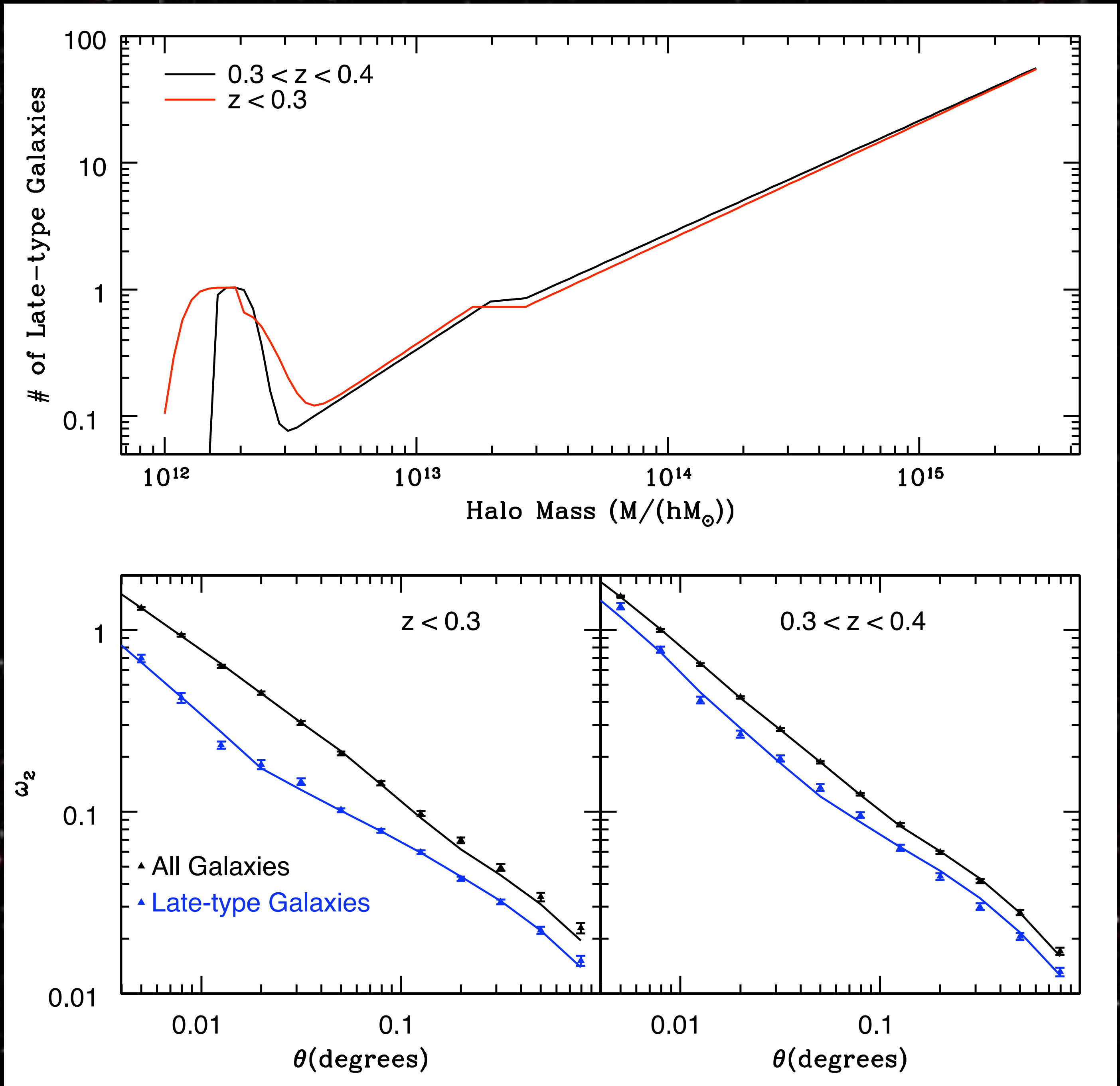
$$\langle N_c \mid M\rangle = 0.5\left(1+\mathrm{erf}\left(\frac{M/M_{cut}}{\sigma_{cut}}\right)\right)$$

$$\langle N_s \mid M\rangle = 0.5\left(1+\mathrm{erf}\left(\frac{M/M_{cut}}{\sigma_{cut}}\right)\right)\times (M/M_0)^{\alpha}$$

$$\langle N_c \mid M\rangle_{late} = \langle N_c \mid M\rangle \times fc_0 \exp\left(\frac{-\log_{10}(M/M_{cut})}{2\sigma_c^2}\right)$$

$$\langle N_s \mid M\rangle_{late} = \langle N_s \mid M\rangle \times fs_0 \exp\left(\frac{-\log_{10}(M/M_0)}{\sigma_s}\right)$$

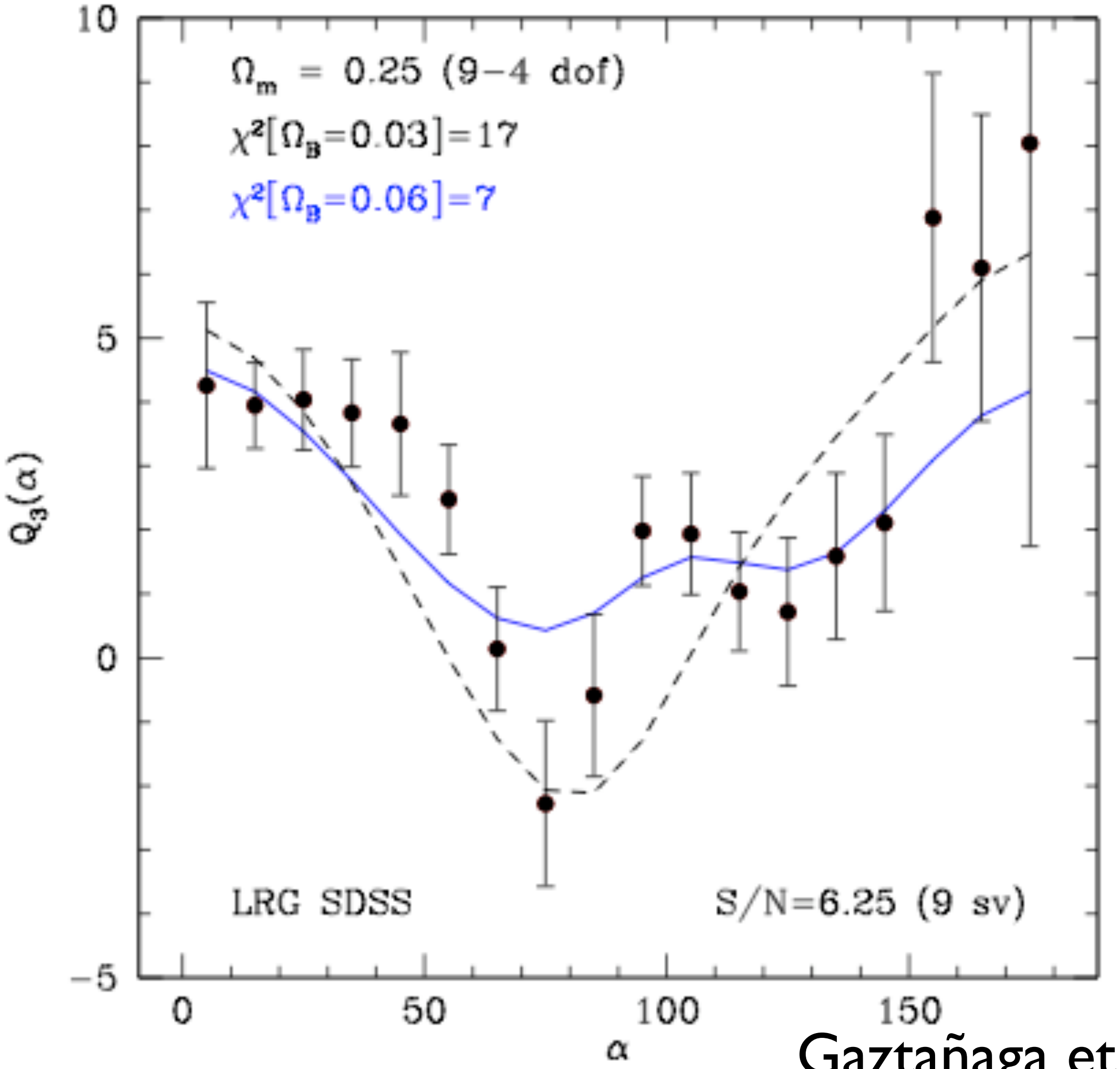
- Results are Preliminary

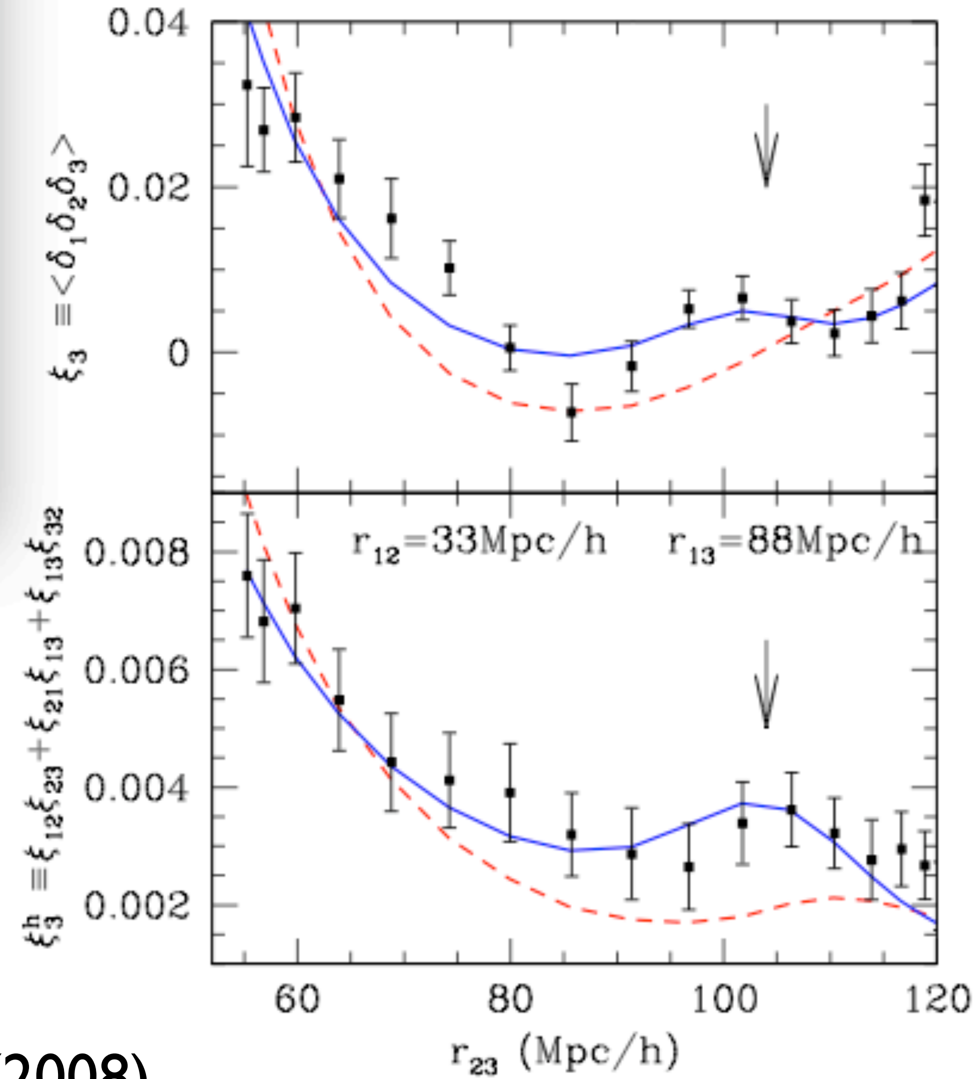


Future Surveys

- Pan-STARRs
  - 3/4 night sky to g ~ 23
  - 1200 deg^2 to z ~ 1.5
  - in calibration mode
- SDSS III - BOSS
  - ~1.5 million LRGs with $0.4 < z < 0.7$
- DES
  - ~5000 deg^2 to z ~ 1.1
- LSST
  - Half the night sky, 10 billion galaxies!
  - (First light 2014)

Future Surveys





Gaztañaga et al. (2008)

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(Hopeful) Future of N^{th}-Order Correlation Functions

- Higher-order measurements become integral
  - Data, computing, human resources exist
  - Combined with 2-point, better constraints, more self-consistency → better science
- Challenges:
  - Urging patience and collaboration
  - Incorporating HOD in higher-order modeling (heavy lifting done in Smith et al. 2008)

To Do

- Talk through out loud